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MODELS FOR CARBON-BURNING STARS*

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ABSTRACT

Equilibrium models are constructed for homogeneous stars containing only C^{12} and heavier elements. An "initial carbon-burning main sequence" is found both with and without neutrino energy loss mechanisms The neutrinos raise the central temperature T_c about 25 per cent, decrease the lifetime by a factor of more than 100, increase the extent of the convective core, decrease the optical luminosity slightly, and raise the minimum mass for carbon burning from 0 7 $M_\odot$ to about 0 8 $M_\odot$. For gravitationally contracting stars, neutrino processes become important when T_c exceeds 0.3 or 0.5×10^9 ° K; they lower the temperature gradient (or even reverse its sign) in the deep interior and the contraction is not homologous.

I. INTRODUCTION

In two preceding papers (Cox and Salpeter 1964; Deinzer and Salpeter 1964; hereinafter referred to as "CS" and "DS," respectively) a series of models was described for helium-burning stars which contain no hydrogen at all. The present paper describes a somewhat similar series of models for chemically homogeneous carbon stars which contain no hydrogen or helium (having previously burned hydrogen to helium and helium to carbon and oxygen) and which derive their energy from the carbon-carbon nuclear reaction. The complete absence of a helium shell and hydrogen envelope for an evolved star is of course unrealistic, but we justify our models on the grounds that (*a*) they are simple enough to make complete series feasible and (*b*) they represent the limiting case for stars which have shed much of their helium shell and hydrogen envelope by mass loss during their evolution.

In carbon-burning stars (unlike hydrogen or helium burning) the internal temperatures are high enough ($> 0.5 \times 10^9$ ° K) so that energy loss due to neutrino processes is important, if the universal Fermi interaction holds for the emission of neutrino pairs. This is likely to be the case but not yet certain, and we construct two series of otherwise identical models, one with and one completely without the neutrino energy loss. In previous papers (Reeves 1963; Stothers 1963) it was already shown that the most

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dramatic effect of neutrino losses is a drastic shortening of the time scale of the carbon-burning evolutionary phase. The main aim of the present paper is to investigate the effect of neutrino losses on the structure of the stellar model, e.g., on optical luminosity, central temperature, size of the convective core, and the minimum mass for which carbon burning is possible. We also construct some models (with and without neutrinos) which represent (in a highly simplified way) gravitationally contracting stars with central temperatures a fraction of 10^{9} ° K.

The assumptions and methods used are discussed in § II. The results are presented in § III for carbon-burning stars and in § IV for gravitationally contracting stars.

II. CONSTRUCTION OF MODELS

The stellar-structure equations for all carbon-burning stars were solved by methods discussed by DS. All integrations were carried out on an electronic computer at the Goddard Institute for Space Studies. Radiation pressure was taken into account fully and the same assumptions (and notations) used as by DS, except as follows.

Sampson (1959) has calculated the change in the electron-scattering opacity κ from the non-relativistic value $\kappa_0 = 0.20\,(1 + X)$ at high temperatures. This change is due to the deviations of Compton scattering from Thomson scattering for energies approaching the electron's rest mass energy ($5\,k \times 10^{9}$ ° K). For temperature $T \lesssim 1.5$ (we measure temperature in units of 10^{9} ° K throughout) Sampson's results can be approximated fairly well by the empirical relation

$$\kappa = \kappa_0\,(1 + 2.2T)^{-1}\,. \tag{1}$$

We neglect electron conduction and the effect of degeneracy on κ, which means that we underestimate optical luminosities slightly for the stars of lowest mass.

Non-relativistic partial degeneracy in the equation of state was taken account of approximately as follows. Let Π be the dimensionless degeneracy parameter proportional to $P_{\text{gas}}T^{-5/2}$ defined in equation (2) of CS. Let Λ be the correction factor in the equation of state,

$$\rho = P_{\text{gas}}[AH/(Z + 1)kT]\Lambda\,, \tag{2}$$

where Z and A are atomic charge and weight of the positive ions. For $\Pi < 7.7$ we use the approximation

$$1 - \Lambda = \frac{0.1768[Z/(Z+1)]\Pi + 3.88 \times 10^{-3} \times \Pi^{2.25}}{1 + [0.3722 - 0.3536(Z+1)^{-1}]\Pi + 3.88 \times 10^{-3} \times \Pi^{2.25}}\,. \tag{3}$$

The terms linear in Π give the correct first-order expansion term for $\Pi \ll 1$ for any Z and A. The terms in $\Pi^{2.25}$ are purely empirical corrections which give Λ accurate to better than 2×10^{-3} for C^{12} and heavier ions. For $\Pi > 7.7$ we use instead another empirical approximation,

$$\Lambda = 1.6596\Pi^{-0.4} - 1.0432\Pi^{-1}\,. \tag{4}$$

Relativistic corrections to Λ were not included, although they are quite appreciable at the highest densities considered ($\sim 10^{6}$ gm/cm^{3}). In equations (2)–(4) we used $Z = 6$ and $A = 12$ throughout our models (the error due to our omission of the heavier elements in the equation of state is not very great).

For the rate of nuclear-energy production ϵ_{nuc} we took the values of Reeves (1963),

$$\begin{aligned} \epsilon_{\text{nuc}} &= 10^{44.7}\rho x_{C}^{2}T^{-2/3}f \exp\left[-84.2\,(1 + 0.1T)^{1/3}T^{-1/3}\right] \text{erg/gm/sec}\,, \\ f &= \exp\,(3.8 \times 10^{-4}\rho^{1/2}T^{-3/2})\,. \end{aligned} \tag{5}$$

For the sake of uniformity we used $x_C = 0.5$ in equation (5) for all stellar masses, even though (see DS, Fig. 3) helium burning results in less C^{12} and more O^{16} for the more massive stars (see note added in proof on p. 822). For the rate of energy loss ϵ_ν due to neutrino pair production we use

$$\epsilon_\nu = \left[10^{7.7} T^8 + 10^{18.7} \frac{T^3}{\rho} e^{-11.9/T} + \frac{\rho^2 T^3}{10^6 + \rho} \exp\left(-\frac{1.7 \rho^{1/3}}{10^3 T} \right) \right] \text{erg/gm/sec}, \quad (6)$$

with ρ the density in gm/cm^3 (and T in units of 10^9 ° K). The first two terms represent the photoneutrino and the electron pair processes, respectively, evaluated for non-degenerate electrons. The third term is a rough empirical approximation to the plasma neutrino rate (Adams, Ruderman, and Woo 1963) for densities and temperatures relevant to our models. The neglect of degeneracy in the first two terms is not very serious since the third term (which includes degeneracy) dominates at the highest densities (see Fig. 1 of Reeves 1963).

For the carbon-burning stars we took for the total rate of energy production $\epsilon = \epsilon_{\text{nuc}} - \epsilon_\nu$ for the one series of models and simply $\epsilon = \epsilon_{\text{nuc}}$ for the other. As described by DS, the central value of the radiation pressure parameter was kept fixed for one model and T_c was considered as the eigenvalue to be found by trial and error. We evaluated

$$L(r) = 4\Pi \int_0^r \epsilon(s) \rho(s) s^2 ds, \qquad L_{\text{nuc}}(r) = 4\Pi \int_0^r \epsilon_{\text{nuc}}(s) \rho(s) s^2 ds \quad (7)$$

and used $L(r)$ in the radiative heat-flow equation.

As a crude model for gravitationally contracting stars we simply assumed a law of gravitational energy release of form

$$\epsilon_{\text{gr}} = AT \text{ erg/cm/sec} \quad (8)$$

with A a constant. This form is a good approximation if the contraction is close to being homologous with the rate of energy release proportional to the heat content of the electrons and ions. This approximation is not accurate when the electrons are fairly degenerate and breaks down when neutrino processes become dominant (see § IV). We took $\epsilon = \epsilon_{\text{gr}} - \epsilon_\nu$ for one series of models and $\epsilon = \epsilon_{\text{gr}}$ for the other. Each series is characterized by two parameters, β_c and T_c. With these two parameters fixed, the constant A in equation (8) was considered as the eigenvalue and otherwise the same methods of integration were used as before.

For the carbon-burning stars a slight correction for gravitational energy release was also estimated as follows. In addition to the carbon-burning models with $x_C = 0.5$ in equation (5), some models were constructed with $x_C = 0.4$. By comparing the total energy content of the initial and the additional model we obtain the gravitational energy released during the burning of 0.1 of the convective core mass from C^{12} to heavier elements. The known energy content of this nuclear fuel and the value of L_{nuc} give the time duration of this evolutionary step, which finally leads to a value for the rate of gravitational energy release (assumed to be of the form of eq. [8]). Another series of models was then constructed with this small and known rate of energy production added to $\epsilon_{\text{nuc}} - \epsilon_\nu$, and T_c was again considered as the eigenvalue.

III. RESULTS FOR CARBON BURNING

The main results for the models for carbon-burning stars are given in Table 1. The notation is described in more detail in CS and DS with β_c, Π_c, T_c, and ρ_c the radiation pressure parameter, degeneracy parameter, temperature, and density (in gm/cm^3) at the center, M and R mass and radius in solar units, L the optical (bolometric) luminosity and L_{nuc} the total rate of nuclear energy production (which equals L plus the neutrino luminosity L_ν) in solar units.

Consider first the series of models without any neutrino energy loss. The mathematical properties of this series are rather similar to those of the homogeneous helium-burning series of models in CS and DS, which are affected by degeneracy at the low-mass end of the series and by radiation pressure at the high-mass end: All the energy production takes place inside the convective core; the fractional mass in the core q_f and the density ratio $\rho_c/\bar{\rho}$ increase monotonically with decreasing β_c; the dimensionless temperature parameter t_c and the luminosity parameter C both have maxima in the middle of the series. The present series of models has temperatures higher by factors of order 10 which also have the following effects. The middle region of mass in which both radiation pressure and degeneracy are unimportant is of smaller extent. According to equation (1) the opacity decreases slightly toward the center, which decrease tends to decrease q_f somewhat and to increase $\rho_c/\bar{\rho}$ and C (see also Boury 1964). The series has a minimum mass for which carbon burning is possible, $M_{\min} = 0.697$. The unlabeled solid curve in Figure 1 plots the ratio L/M^3 (which would be constant for the perfect gas law with a constant opacity) against mass M (all in solar units).

TABLE 1

PROPERTIES OF CARBON-BURNING MODELS*

β_c	Π_c	log T_c	ρ_c $\times 10^{-4}$	q_f	M	100 $R/R_\odot$	L	L_{nuc}	$\rho_c/\bar{\rho}$	p_c	t_c	$-$log C	log T_{eff}
Without Neutrinos													
0 9975	10 2	8 776	241	0 260	0 707	1 58	54 6		...	...	...	...	...
995	5 40	8 794	165	257	0 705	1 95	110		12 26	24 5	0 437	3 744	5 13
99	2 87	8 813	107	258	0 775	2 48	247		15 03	31 9	530	3 515	5 16
98	1 51	8 832	67 0	262	0 925	3 29	614		18 29	40 9	614	3 351	5 20
95.	0 64	8 857	33 8	29	1 34	5 00	2 46(3)		22 5	53 1	685	3 228	5 26
.875	0 26	8 884	15 8	363	2 35	8 22	1 33(4)		26 3	63 7	680	3 229	5 34
75	0 120	8 907	8 08	487	4 55	13 34	6 65(4)		30 1	74 1	604	3 391	5 40
6	0 064	8 925	4 61	608	9 27	22 0	2 65(5)		37 8	97 1	509	3 718	5 45
0 4	0 030	8 944	2 35	0 739	27 0	50 5	1 35(6)		79 7	251	0 418	4 405	5 45
With Neutrinos													
0 996	9 38	8 889	339	0 809	0 834	1 39	34 7	2 80(5)	...	...	...	...	...
993	5 51	8 898	240	760	0 820	1 62	74 4	3 05(5)	8 81	16 1	0 397	4 111	5 12
99	3 93	8 905	189	732	0 844	1 83	124	3 43(5)	9 73	18 4	442	3 925	5 15
98	2 04	8 917	116	682	0 961	2 40	350	4 94(5)	11 80	23 6	525	3 644	5 21
95.	0 84	8 937	57 4	645	1 333	3 67	1 61(3)	1 12(6)	15 19	32 5	607	3 407	5 28
875	0 34	8 963	26 8	644	2 31	6 20	1 05(4)	4 78(6)	19 67	44 3	626	3 313	5 37
75	0 161	8 992	14 4	685	4 51	10 35	6 22(4)	3 41(7)	25 4	59 8	575	3 409	5 45
6 .	0 089	9 022	8 94	732	9 26	17 4	2 74(5)	2 83(8)	36 3	92 6	503	3 701	5 50
0 4	0 045	9 060	5 20	0 775	27 1	48 8	1 48(6)	4 83(9)	158	629	0 525	4 371	5 46
With Neutrinos and Gravitation													
0 993	5 51	8 898	239	0 660	0 835	1 90	310	3 04(5)	13 9	29 3	0 457	3 514	5 24
99	3 93	8 904	189	636	0 862	2 15	432	3 42(5)	15 5	33 8	510	3 412	5 25
98	2 03	8 917	116	598	0 985	2 83	918	4 94(5)	18 9	43 5	604	3 258	5 28
95	0 84	8 937	57 3	570	1 368	4 30	3 18(3)	1 12(6)	23 6	57 3	690	3 146	5 32
875	0 34	8 962	26 8	560	2 39	7 50	1 88(4)	4 77(6)	33 6	88 3	731	3 102	5 39
0 75	0 161	8 992	14 4	0 520	4 94	19 0	1 39(5)	3 40(6)	142	561	0 962	3 178	5 41

* See note added in proof on p 822

Consider next the carbon-burning models with neutrino energy loss included. The effect of the neutrinos is quite strong in the sense that the neutrino luminosity L_ν is very much larger than the optical luminosity L. The nuclear energy output L_{nuc} is then very close to L_ν, and L_{nuc}/L is plotted against stellar mass in the dashed curve in Figure 1. This factor exceeds 400 for all masses, and the time scale for carbon burning is shortened by almost the same factor due to neutrino processes. The burning time required to decrease x_{C} from 0.5 to 0.4 for $M = 0.82$, 1.33, 4.5, and 27.1, for instance, would only be 1800, 680, 80, and 4 years. The high values required for L_{nuc} also lead to substantially higher internal temperatures. In Figure 2 the central temperature is plotted against stellar mass, the lower curve for models without and the top curve for models

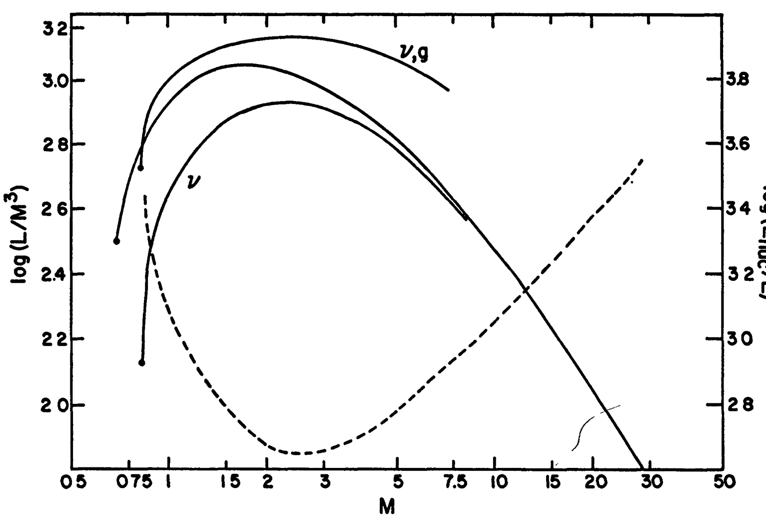

FIG. 1.—The ratios L/M^3 and L_{nuc}/L (*dotted curve*) plotted against M for carbon-burning models, where L is optical luminosity, L_{nuc} is total nuclear energy production, and M is mass (all in solar units). The unlabeled solid curve is without neutrinos; curve ν, with neutrinos; and curve ν, g, with neutrinos and a gravitational correction.

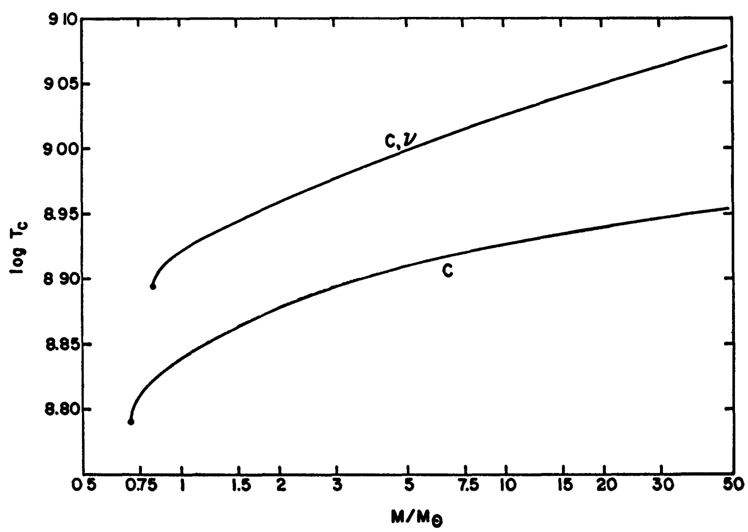

FIG. 2.—Central temperature T_c plotted against mass M for carbon-burning models without (C) and with (C, ν) neutrino processes.

with neutrino energy loss. The minimum mass for carbon burning is also larger, $M_{\min} = 0.820$, because higher temperatures are required.

The neutrino processes also affect other properties of the models. The nuclear energy production rates are much more temperature-sensitive than the neutrino energy loss rates, so that the former dominates in the deep interior and the latter dominates further out. The optical heat flux $L(r)$ through a spherical shell of radius r is plotted against the mass $M(r)$ contained inside r in Figure 3 for models with $\beta_c = 0.95$. $L(r)$ has a sharp maximum for the model with neutrinos (labeled "C, ν") in contrast with the familiar monotonic rise for models without (labeled "C"). The large heat flux in the interior regions greatly increases the fractional mass q_f in the convective core. The temperature-density variation (from the center outward) for the models with $\beta_c = 0.95$ is shown in Figure 4. The decrease of $L(r)$ with r in the outer regions also leads to slightly lower values of L (for fixed β_c or for fixed M) as Figure 1 shows. These effects all become less

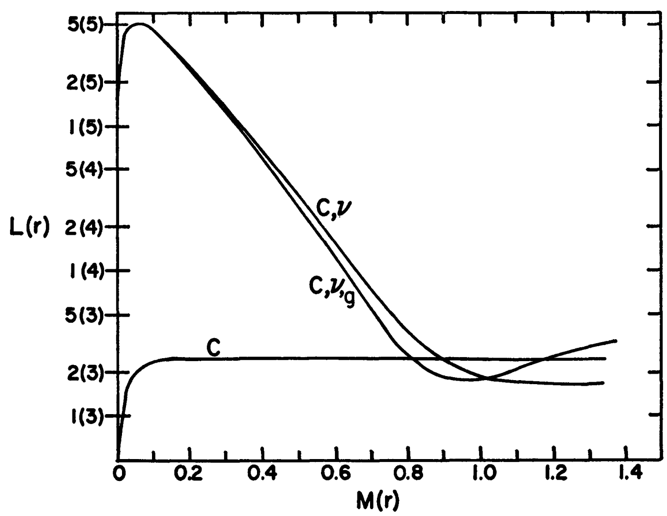

FIG. 3.—Optical radiation flux $L(r)$ plotted (with powers of 10 in parentheses) against internal mass $M(r)$ for carbon-burning models without (C) and with neutrinos (C, ν) and with both neutrinos and a gravitational correction (C, ν, g).

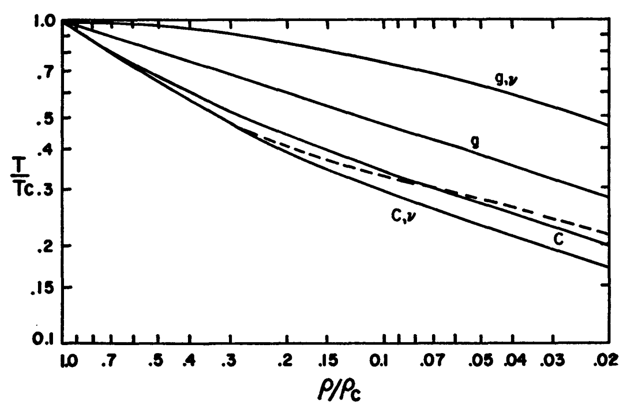

FIG. 4.—Temperature T plotted against density ρ (in terms of central values) for carbon-burning with (C, ν) and without (C) neutrinos (*dotted curve* with neutrinos and gravitation); also for gravitationally contracting models with (g, ν) and without (g) neutrinos.

marked for large stellar masses where q_f is large even in the absence of neutrino processes.

We finally turn to carbon-burning stars with neutrino loss for which a small amount of gravitational energy release has been added, using equation (8) as described in § II. The ratio of the gravitational to nuclear energy release for the whole star was fairly small, between 0.011 and 0.012 for all masses (see note added in proof on p. 822). This energy release is nevertheless important since it is larger than L and occurs at relatively large radial distances because of the very low temperature sensitivity of equation (8). In Figure 3 $L(r)$ is compared for the three types of model, each with $\beta_c = 0.95$. Note that the three curves all cross at intermediate values of $M(r)$. As Figure 1 shows, the optical luminosity (for fixed mass) is largest for the models with the gravitational correction (labeled "C, ν, g" in Fig. 3) because of the rise of $L(r)$ near the outside for such models.

IV. RESULTS FOR GRAVITATIONAL CONTRACTION

We are dealing here with a two-parameter family of models (T_c and β_c), and we evaluated series of models for $T_c = 0.3$, 0.4, and 0.5. Results are presented in Table 2 for

TABLE 2

PROPERTIES OF GRAVITATIONALLY CONTRACTING MODELS

β_c	Π_c	$10^{-3}\ \rho_c$	M	100 $R/R_\odot$	L	L_{pos}	τ_{gr}	$\rho_c/\bar{\rho}$	p_c	t_c	$-\log C$	$\log T_{\text{eff}}$
T_c=0 3×10⁹ ° K without Neutrinos												
0 9995	18 1	1,211	0 499	1 80	21 2		3 0(6)	9 97	10 0	0 274	4 008	4 97
999	9 07	798	0 454	2 27	35 9		1 93(6)	14 73	29 8	383	3 657	4 97
99	0 90	124	0 718	6 55	493		2 9(5)	34 8	82 5	698	3 116	5 03
98	0 44	64 7	0 963	9 4	1 29(3)		1 52(5)	39 5	95 6	744	3 080	5 05
95	0 172	26 0	1 538	15 2	5 08(3)		6 3(4)	41 8	101 8	752	3.095	5 10
75	0 027	4 19	5 22	41 0	8 60(4)		1 27(4)	39 2	94 2	598	3 459	5 19
6	0 014	2 10	9 97	61 0	2 65(5)		8 0(3)	34 0	79 7	467	3 814	5 22
0 4	0 001	0 93	26 73	105	1 09(6)		5 2(3)	28 6	64 4	0 299	4 482	5 26
T_c=0 3×10⁹ ° K with Neutrinos												
0 9975	3 62	405	0 819	7 0	1 27(3)	2 10(3)	9 9(4)	120 5	327	0 652	2 878	5 12
995	1 81	229	0 796	7 1	9 26(2)	1 30(3)	1 41(5)	74 1	186 4	684	2 977	5 08
99	0 90	124	0 872	8 3	1 10(3)	1 34(3)	1 41(5)	58 8	146 0	731	3 022	5 06
0 95	0 172	26 0	1 623	16 5	6 03(3)	6 35(3)	5 4(4)	51 3	129 1	0 777	3 091	5 10
T_c=0 5×10⁹ ° K without Neutrinos												
0 95	0 37	117	1 645	9 8	6 78(3)		8 3(4)	47 6	119 9	0 757	3 058	5 22
0 75	0 059	19 3	5 58	28 3	1 12(5)	.	1 72(4)	55 7	145 0	0 645	3 430	5 30
T_c=0 5×10⁹ ° K with Neutrinos												
0 85	0 111	36 2	19 4	12 1	8 25(5)	1 00(6)	5 7(3)	2,330	6,614	0 791	4 187	5 20
0 75	0 059	19 3	27 2	140	1 26(6)	1 47(6)	5 4(3)	1,380	3,641	0 653	4 445	5.21

only a fraction of the models evaluated; τ_{gr} gives the time scale for contraction in years; L_{pos} is $L + L_\nu$. For the models with $T_c = 0.5$ and $\beta_c = 0.75$ we also plot temperature $T(r)$ against density $\rho(r)$ in Figure 4 and heat flux $L(r)$ against mass $M(r)$ inside radial distance r in Figure 5.

Consider first the models with pure gravitational energy release without neutrino processes (labeled "g" in Figs. 4 and 5). The dominant feature of such models (compared to models involving nuclear fuel) is the weak temperature sensitivity and hence the large spatial extent of the energy source, i.e., the slow rise of $L(r)/L$ in Figure 5 compared with Figure 3. As a consequence the ratio of temperature-to-density gradient $d \ln T/d \ln \rho$ in Figure 4 is appreciably smaller in the deep interior than for nuclear-burning models. For models where neither radiation pressure nor degeneracy are overwhelming (such as the one in Figs. 4 and 5) $d \ln T/d \ln \rho$ and other mathematical model properties are qualitatively similar to those for the $n = 3$ polytrope.

For models with $T_c = 0.3$, the minimum mass was found to be $M_{\min} = 0.453$. As discussed by CS (in connection with helium burning), $M_{\min}$ scales roughly as $T_c^{3/4}$ so that $M_{\min}$ $(T_c = 0.6) \approx 0.76$ for our models with extended energy sources compared

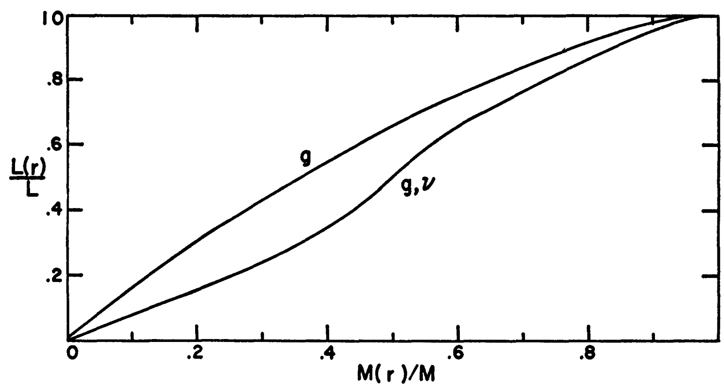

FIG. 5.—Radiation flux $L(r)$ plotted against internal mass $M(r)$ for gravitationally contracting models with (g, ν) and without (g) neutrinos.

with the minimum mass of 0.70 for pure carbon-burning models with $T_c \approx 0.61$. In the absence of neutrino processes (and without relativity) the actual minimum mass for carbon burning is likely to lie between 0.70 and 0.77 $M_\odot$. Relativistic corrections to the degeneracy formulae will lower the numerical values for $M_{\min}$, however.

We turn now to the gravitationally contracting models *with* neutrino energy loss (labeled "g, ν" in Figs. 4 and 5). In contrast to carbon-burning stars, the neutrino-energy loss rate has a *stronger* temperature dependence than the energy production rate. As Figure 5 shows, the neutrinos thus have the effect of making $L(r)$ increase less rapidly with r and hence (see Fig. 4) of decreasing the temperature gradient $d \ln T/d \ln \rho$. For some of the models (with larger ρ_c than the illustrated model), the temperature $T(r)$ actually *increases* slightly with increasing r in the deep interior. This stems from the fact that the plasma neutrino rates, which dominate at high densities, have a stronger density dependence than the gravitational energy release. Thus energy released in intermediate regions of the star partly flows inward to be dissipated by the plasma neutrinos near the center.

The flat temperature distribution for our gravitation-neutrino models leads (for fixed T_c and β_c) to larger masses and larger $\rho_c/\bar{\rho}$. The minimum mass at $T_c = 0.3$, for instance, was $M_{\min} = 0.795$ (compared with 0.453 without neutrinos, both without relativistic corrections). At higher temperatures the neutrino processes had the following even more dramatic effects on the models.

For the gravitation-neutrino models listed in Table 2 the neutrino luminosity $L_\nu = L_{\text{pos}} - L$ is less than the optical luminosity L. For our present ranges of ρ and T the ratio L_ν/L increases with T_c and, for fixed T_c, increases with ρ_c and hence with decreasing mass. For combinations of large enough values of T_c and ρ_c we found that the outward integrations of the stellar-structure equations with neutrinos included did not give any convergent models at all. Mathematically, the divergence stems from the neutrino-induced reversed temperature gradient in the deep interior coupled with the very extended energy source implied by equation (8). Physically, the situation for a contracting star of given mass is simply that the contraction ceases to be close to homologous as soon as the neutrino processes become dominant. For correct evolutionary models the factor A in equation (8) is then a decreasing function of radial distance r instead of a constant. These effects of non-homology are not very severe if $T_c \lesssim 0.3$, 0.4, and 0.5, respectively, for masses $M = 0.8$, 1.8, and 20.

V. DISCUSSION

We have presented models for homogeneous stars consisting largely of carbon and oxygen and containing no hydrogen and helium at all. If neutrino processes are assumed to be non-existent, we found that a minimum mass of about 0.7 $M_\odot$ is required for carbon burning to be possible. The mathematical properties for such an "initial carbon-burning main sequence" are similar to those for helium-burning models but central temperatures are in the range $T_c = 0.6$–1.0×10^9 ° K. The gravitational contraction prior to carbon burning is at least roughly homologous.

We have investigated the effects of neutrino energy loss on these stellar models, combining the rates for the plasma neutrino process (dominant at the highest densities), for photo-neutrino production and for the electron-pair neutrino process (dominant at the highest temperatures). For the carbon-burning models the main effects were as follows. Central temperatures T_c are increased by typically 25 per cent (see Fig. 2). The total optical flux $L(r)$ crossing a sphere of radius r is not monotonic but has a strong maximum at finite r (Fig. 3). The optical luminosity L is decreased slightly (by a factor of less than 2), the neutrino luminosity $L_\nu = L_{\text{nuc}} - L$ is larger than L by a factor of more than 100 (Fig. 1) and the lifetime of the carbon-burning stage is reduced by similar factors. The temperature gradient in the interior (Fig. 4) and the fractional mass of the convective core are increased.

For models with a very extended energy source such as gravitational contraction the main neutrino effects are: For given T_c and ρ_c the mass is increased and for sufficiently large T_c and ρ_c the contraction is not even approximately homologous. The minimum mass, required to reach a given maximum central temperature, is increased. The heat flux $L(r)$ in the deep interior is decreased (Fig. 5) or (for models with large ρ_c) even negative, and the temperature gradient (Fig. 4) is also decreased or reversed.

If a star has previously suffered appreciable mass loss from its hydrogen- and helium-rich outer layers (subsequent to helium burning but before carbon burning) its evolution may approximate that given by our homogeneous model (except that even a small hydrogen envelope will increase radius and decrease T_c with little effect on luminosity). For a star of given mass M, the effect of neutrino processes on the evolution is then as follows. For $M \lesssim 0.45\ M_\odot$ the effects are rather slight, since the maximum temperature T_c reached is below 0.3×10^9 ° K and the neutrino luminosity is low. For M between about 0.45 $M_\odot$ and 0.75 $M_\odot$ the carbon-burning stage is not reached in either case, but $T_{c,\max}$ is lower with neutrinos (about 0.3 instead of 0.6×10^9 ° K for 0.75 $M_\odot$) and the gravitational contraction time is shortened. For a small range of masses just above 0.75 $M_\odot$ carbon burning would be reached in the absence of neutrinos, but not with them. For stars exceeding about 1 $M_\odot$ the carbon-burning stage is reached in either case but last for a much shorter period with neutrino processes. For very massive stars the composition is likely to be largely O^{16} and rather little C^{12}, and (with neutrinos) gravitational contraction affects the optical luminosity and structure even during the carbon-burning stage. The models presented in the present paper are not suitable in such cases, and genuine evolutionary models will have to be constructed.

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Note added in proof: The computations of models in Table 1 were inadvertently carried out for lower values of the carbon abundance x_C than stated in the text. The entries in Table 1 labeled "Without" and "With" are self-consistent models for approximately the following abundances:

	"Without"				"With"			
M. ...	0 77	1 34	4 55	27 0	0 84	1 33	4 51	27 1
100 x_C	2 0	2 1	2 3	2 6	2 4	2 5	2 7	3 1

The models labeled "With Neutrinos and Gravitation" in Table 1 are incorrect, however, because values of about 0.012 (appropriate for $x_C = 0.5$), instead of the appropriate values of about 0.3 (for $x_C \sim 0.02$ to 0.03), were used in the correction for gravitational energy release. Correcting this error would raise L and R and lower q_f, but probably have little effect on central conditions.

For the most massive stars on an initial carbon-burning main sequence, values of x_C as low as those above may be appropriate, but not for the lower masses. Evolutionary models with $x_C = 0.5$ for the lowest masses will be presented shortly. For the intermediate and larger masses we have made the following estimates (assuming homologous changes) for the amounts by which various quantities in Table 1 would change (+ denotes an added term, $\times$ a multiplying factor) if the abundance were changed to $x_C = 0.5$:

	Without			With		
M.	1 34	4 55	27 0	1 33	4 51	27 1
log T_c	−0 08	−0 08	−0 08	−0 11	−0 13	−0 14_5
ρ_c .	×0 6	×0 6	×0 6	×0 5	×0 4	×0 3_5
L_{nuc}	1	1	1	×0 11	×0 035	×0 013

For the models without neutrinos in the mass range above, the assumption of homologous change is probably reasonable and the other quantities in Table 1 should not change very much. For the models with neutrinos the lowered values of $L_\nu \approx L_{nuc}$ (for the larger value of x_C) will lower q_f and raise L. With the lowered values of L_{nuc}, the corrections for gravitational energy release will be much smaller than indicated by Table 1 and can probably be neglected.

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